

and β_2 is obtained from α_2 by reversing the sign of γ . It is obvious from (41) that β_2 is much larger than α_2 for $\lambda/\gamma \ll 1$. For this reason an expansion of R in powers of $\delta\omega$ converges very slowly, but an expansion of $1/R$ in powers of $\delta\omega$ should converge rapidly. If higher powers of $\delta\omega$ are again neglected, (14) is obtained from (40) and

$$\left(\frac{2}{B}\right)^2 = \beta_2 - \alpha_2. \quad (42)$$

A simple expression for the bandwidth is obtained in the

limit $\lambda/\gamma \ll 1$. The α_2 term of (42) can be neglected if only the lowest order of λ/γ is taken into account. In this approximation

$$B^2 = \frac{4}{\beta_2} = \frac{2(\lambda/\gamma)^2 \omega_0^2}{1 + \frac{1}{a^2} + \frac{1}{2a^4} + \left(1 + \frac{1}{a^2}\right) \frac{2\pi M}{H + 2\pi M}}. \quad (43)$$

Eq. (15) is now obtained by expressing H in terms of the resonance frequency.

Analysis of Microwave Measurement Techniques by Means of Signal Flow Graphs*

J. K. HUNTON†

Summary—Microwave measurement techniques can be analyzed more simply by using signal flow graphs instead of the customary scattering matrices to describe the microwave networks used in the measuring system. This is because the flow graphs of individual networks are simply joined together when the networks are cascaded and the solution for the system can be written down by inspection of the over-all flow graph by application of the nontouching loop rule. This paper reviews the method of setting up flow graphs of microwave networks and the rule for their solution. A single directional-coupler reflectometer system for measuring the reflection coefficient of a load is then analyzed by this method. The analysis shows how auxiliary tuners can be used to cancel residual error terms in the measurement of the magnitude of the reflection coefficient at a particular frequency. The analysis also shows how an additional tuner can be used to measure the phase angle of the reflection coefficient. These reflectometer techniques are particularly useful in the measurement of very small reflections.

INTRODUCTION

THE signal flow graph is a method of writing a set of equations, whereby the variables are represented by points and the interrelations by directed lines giving a direct picture of signal flow. The algebra of flow graphs leading to solutions by direct inspection has been developed by S. J. Mason and others at the Massachusetts Institute of Technology.^{1,2} When microwave network equations are written in scattering matrix form the corresponding flow graph is particularly useful because, in this case, the flow graph of a system of cas-

caded networks is constructed simply by joining together the flow graphs of the individual networks, and the solution is then available directly.

One of the best applications of the flow graph method is in the analysis of measuring techniques and the determination of residual errors. It is the intention here to review the mechanics of the method and to apply it in analyzing the microwave reflectometer system used for measuring the reflection coefficient of a load. This system has been in general use for some time,³ and has been analyzed recently by Engen and Beatty⁴ who showed how tuners could be used to reduce residual errors to a negligible value when measuring the magnitude of the reflection coefficient. Their result will be derived here by the flow graph method. In addition, a technique for measuring the phase angle of the reflection coefficient will be presented.

ONE- AND TWO-PORT NETWORK FLOW GRAPHS

Fig. 1 shows some simple flow graphs used as building blocks. In Fig. 1(a) the general two-port network is shown as specified by its scattering matrix coefficients. Here a_1 and a_2 are the complex entering wave amplitudes, while b_1 and b_2 are the outgoing wave amplitudes at ports 1 and 2 of the network. These are represented in the flow graph as points or "nodes." The nodes are

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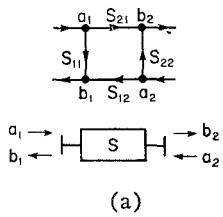
† Hewlett-Packard Co., Palo Alto, Calif.

¹ S. J. Mason, "Feedback theory—some properties of signal flow graphs," *PROC. IRE*, vol. 41, p. 1144-1156; September 1953.

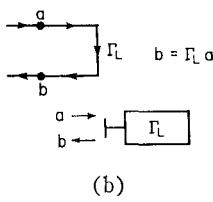
² S. J. Mason, "Feedback theory—further properties of signal flow graphs," *PROC. IRE*, vol. 44, pp. 920-926; July, 1956.

³ J. K. Hunton and N. L. Pappas, "The -hp- microwave reflectometers," *Hewlett-Packard J.*, vol. 6, pp. 1-7; September-October; 1954.

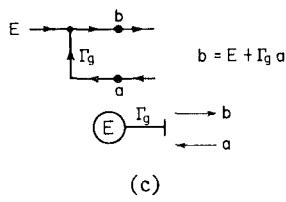
⁴ G. F. Engen and R. W. Beatty, "Microwave reflectometer techniques," *IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES*, vol. MTT-7, pp. 351-355; July 1959.



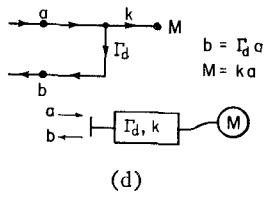
(a)



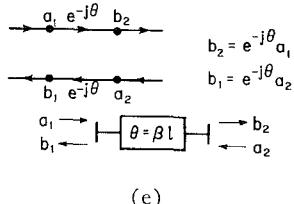
(b)



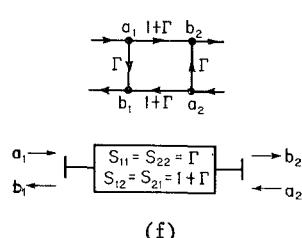
(c)



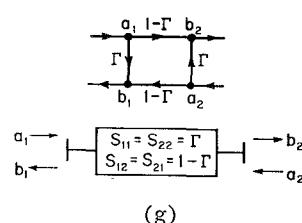
(d)



(e)



(f)



(g)

Fig. 1—(a) Two-port network. (b) Load. (c) Generator. (d) Video detector. (e) Lossless line length. (f) Shunt admittance, (g) Series impedance.

related to one another by directed lines (signal flow) marked with appropriate coefficients. These are the scattering coefficients S_{11} , S_{12} , S_{21} , S_{22} and their meaning is derived from

$$b_1 = S_{11}a_1 + S_{12}a_2,$$

$$b_2 = S_{21}a_1 + S_{22}a_2.$$

Here S_{11} is the reflection coefficient b_1/a_1 at port 1 when port 2 is terminated in a matched load (in this case $a_2=0$). S_{22} is the reflection coefficient b_2/a_2 at port 2 when port 1 is matched ($a_1=0$). S_{12} is the transmission coefficient b_1/a_2 from port 2 to port 1 when port 1 is matched ($a_1=0$), and S_{21} is the transmission coefficient b_2/a_1 from port 1 to port 2 when port 2 is matched ($a_2=0$). In all reciprocal networks $S_{12}=S_{21}$. The value of each node in the flow graph is the sum of all signals entering it, each signal being the value of the node from which it comes multiplied by its path coefficient. The independent variables a_1 and a_2 in the equations represented by the flow graph are characterized by signal flow directed into the graph.

Fig. 1(b) depicts a termination or load whose reflection coefficient is Γ_L .

Fig. 1(c) shows a mismatched generator. Here E is the wave amplitude at the port when the generator sees a matched load ($a=0$) and Γ_g is the reflection coefficient looking into the port when E is zero.

Fig. 1(d) shows a video detector (such as a crystal or a barretter mount). Γ_d is the detector reflection coefficient at the port, and k is a scalar conversion efficiency relating the incoming wave amplitude to a meter reading M . It is assumed that this meter is calibrated to take account of the detector law so that k is independent of signal level. It is also assumed that Γ_d is independent of signal level. (Both these conditions are satisfied very nearly with detectors used in reflectometer systems when used in their proper operating range.)

Fig. 1(e) depicts a length of lossless transmission line.

Fig. 1(f) is a shunt discontinuity such as a junction between two lines or a probe which can be considered as a shunt admittance. The coefficient $S_{11}=S_{22}=\Gamma$ is the reflection coefficient which would be measured if the discontinuity were followed by a matched load. The coefficient $S_{12}=S_{21}=1+\Gamma$ follows from the fact that the net wave amplitudes on either side of the discontinuity must be equal. The coefficient Γ is related to the normalized shunt admittance Y by

$$\Gamma = -\frac{Y}{Y+2}.$$

Fig. 1(g) is a lumped series impedance. Here the coefficient Γ is related to the normalized series impedance Z by

$$\Gamma = \frac{Z}{Z+2}.$$

THE "NONTOUCHING LOOP" RULE

When networks are cascaded it is only necessary to cascade the flow graphs since the outgoing wave from one network is the incoming wave to the next. This is demonstrated in Fig. 2 where a network is placed between a generator and a load. The system now has only one independent variable, the generator amplitude E . The flow graph contains paths and loops. A "path" is a series of directed lines followed in sequence and in the same direction in such a way that no node is touched more than once. The value of the path is the product of all coefficients encountered en route. In the figure there is one path from E to b_2 . It has a value S_{21} . There are two paths from E to b_1 , namely S_{11} and $S_{21}\Gamma_L S_{12}$. A first order "loop" is a series of directed lines coming to a closure when followed in sequence and in the same direction with no node passed more than once. The value of the loop is the product of all coefficients encountered en route. A second-order loop is the product of any two first-order loops which do not touch at any point and a third-order loop is the product of any three first-order loops which do not touch, and so on. In Fig. 2 there are three first-order loops, namely, $\Gamma_g S_{11}$, $S_{22}\Gamma_L$, and $\Gamma_g S_{21}\Gamma_L S_{12}$ and there is one second-order loop $\Gamma_g S_{11} S_{22}\Gamma_L$.

The solution of a flow graph is accomplished by application of the nontouching loop rule,^{5,6} which, written symbolically, is

$$T = \frac{P_1(1 - \sum L(1)^{(1)} + \sum L(2)^{(1)} - \sum L(3)^{(1)} + \dots) + P_2(1 - \sum L(1)^{(2)} + \sum L(2)^{(2)} \dots) + P_3(1 \dots)}{1 - \sum L(1) + \sum L(2) - \sum L(3) + \dots}$$

Here $\sum L(1)$ denotes the sum of all first-order loops. $\sum L(2)$ denotes the sum of all second-order loops and so on. P_1 , P_2 , P_3 , etc., are the values of all the various paths which can be followed from the independent variable node to the node whose value is desired. $\sum L(1)^{(1)}$ denotes the sum of all first-order loops which do not touch path P_1 at any point, and so on. In other words, each path is multiplied by the factor in brackets which involves all the loops of all orders which that path does not touch. T is a general symbol representing the ratio between the dependent variable of interest and the independent variable. This process is repeated for each independent variable of the system and the results are summed.

As examples of the application of the rule, the transmission b_2/E and the reflection coefficient b_1/a_1 are written as follows:

$$\frac{b_2}{E} = \frac{S_{21}}{1 - \Gamma_g S_{11} - S_{22}\Gamma_L - \Gamma_g S_{21}\Gamma_L S_{12} + \Gamma_g S_{11} S_{22}\Gamma_L},$$

$$\frac{b_1}{a_1} = \frac{S_{11}(1 - S_{22}\Gamma_L) + S_{21}\Gamma_L S_{12}}{1 - S_{22}\Gamma_L}.$$

Note that the generator flow graph is unnecessary when solving for b_1/a_1 , and the loops associated with it are deleted when writing this solution. It is worth mentioning at this point that second- and higher-order loops can quite often be neglected while writing down the solution if one has orders of magnitude for the various coefficients in mind.

THREE-PORT NETWORK

The flow graph of the general three-port network with the third port terminated by a detector is shown in Fig. 3(a). The equations described by the flow graph are

$$b_1 = S_{11}a_1 + S_{12}a_2 + S_{13}a_3,$$

$$b_2 = S_{21}a_1 + S_{22}a_2 + S_{23}a_3,$$

$$b_3 = S_{31}a_1 + S_{32}a_2 + S_{33}a_3,$$

$$a_3 = b_3\Gamma_d,$$

$$M = k_{d3},$$

(note also that $S_{12} = S_{21}$, $S_{13} = S_{31}$, $S_{23} = S_{32}$ for reciprocal networks).

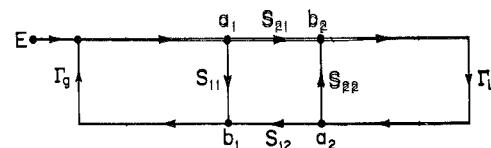
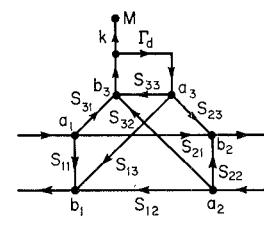
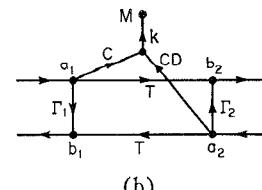


Fig. 2—Cascading of a network between load and generator.



(a)



(b)

Fig. 3—(a) Three-port network with detector. (b) Directional detector.

⁵ C. S. Lorens, "A Proof of the Nonintersecting Loop Rule for the Solution of Linear Equations by Flowgraphs," Res. Lab. of Electronics, M.I.T., Cambridge, Mass., Quart. Prog. Rept., pp. 97-102; January, 1956.

⁶ W. W. Happ, "Lecture notes on signal flowgraphs," from "Analysis of Transistor Circuits," Extension Course, University of California, Berkeley, Catalogue No. 834AB.

Since only two RF ports are available with this combination, the flow graph can be simplified considerably. Fig. 3(b) shows this simplification. The symbols for the coefficients are chosen with a directional coupler-detector combination in mind. The directional coupler is assumed to have a built-in termination in one end of its secondary arm, and the other end of the secondary arm is the third port which is terminated by a video detector. The relationships involved are

$$M = \left| CkE' \frac{(D + T\Gamma_{t1}) + \Gamma_L(TT_t^2 - T\Gamma_{t1}\Gamma_{t2} - D\Gamma_{t2})}{(1 - \Gamma_g'\Gamma_2 - \Gamma_g'\Gamma_{t1}T^2 - \Gamma_1\Gamma_{t1}) - \Gamma_L(\Gamma_g'T^2T_t^2 + \Gamma_1T_t^2 + \Gamma_{t2} - \Gamma_g'T^2\Gamma_{t1}\Gamma_{t2})} \right|.$$

$$b_1 = \Gamma_1 a_1 + T a_2$$

$$b_2 = \Gamma_2 a_2 + T a_1$$

$$M = k(Ca_1 + CDa_2),$$

$$\Gamma_1 = S_{11} + \frac{S_{13}{}^2 \Gamma_d}{1 - S_{33} \Gamma_d},$$

$$\Gamma_2 = S_{22} + \frac{S_{23}{}^2 \Gamma_d}{1 - S_{33} \Gamma_d},$$

$$T = S_{21} + \frac{S_{13}S_{23}\Gamma_d}{1 - S_{33}\Gamma_d},$$

$$C = \frac{S_{31}}{1 - S_{33} \Gamma_d}, \quad D = \frac{S_{32}}{S_{31}}.$$

These relationships are written directly through application of the nontouching loop rule. Note that the path a_1 to M is the main coupling direction involving an effective coupling coefficient C and the path a_2 to M is the residual coupling direction involving the coupling factor and effective directivity coefficient D . For a directional coupler, the coupling factor as usually defined is $20 \log |1/S_{31}|$ while the directivity is $20 \log |S_{31}/S_{32}|$.

SINGLE COUPLER REFLECTOMETER

A reflectometer system for measuring the reflection coefficient of a load is shown in Fig. 4. In this arrangement a single directional-detector is used in conjunction with two slide-screw tuners, one at each end of the coupler. These tuners are for the purpose of cancelling residual signals which can cause a measurement error. They consist of a probe of adjustable penetration projecting into the line through a slot along which the probe position can be varied. In the flow graph of the system

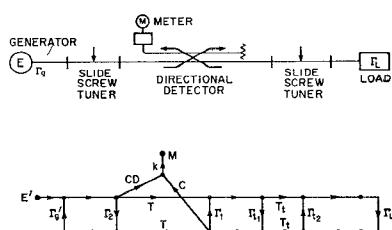


Fig. 4—Single coupler reflectometer.

the generator tuner reflection is lumped together with the generator reflection as Γ_g' and the load tuner is represented as a general two-port network with coefficients Γ_{11} , Γ_{12} and T_L . The analysis carried out in Appendix I shows that Γ_g' can be made equal to any arbitrary value by proper adjustment of the generator tuner (although E varies with the adjustment), and Γ_{11} can be made any arbitrary value by proper adjustment of the load tuner.

The solution for the meter reading M is

This assumes that connector or flange joint reflections are lumped within the tuner networks and the coupler coefficients Γ_1 , Γ_2 , D are small compared to unity. All third- and higher-order loops are negligible and second-order loops involving Γ_1 or Γ_2 are negligible. These approximations are quite valid for practical systems and simplify the algebra considerably. Since the meter reading M is not directly proportional to $|\Gamma_L|$, the reflectometer system as it stands cannot give an accurate result. The procedure for achieving the accurate relationship is as follows:

- 1) Adjust the load tuner: terminate the system with a low-reflection phaseable load. The Γ_L term in the denominator is then negligible by comparison with the constant term, whereas the Γ_L term in the numerator is comparable to the constant term. As the load is moved, the meter reading will vary. By adjusting Γ_{t1} such that no variation occurs, the constant term in the numerator can be brought to zero. This means $\Gamma_{t1} = -D/T$.
 - 2) Adjust the generator tuner: the system is now terminated with a phaseable short circuit. As this is moved, the meter reading varies as a result of the beating between the Γ_L term and the constant term in the denominator. By proper adjustment of $\Gamma_{g'}$, the Γ_L term can be made zero. That is

$$\Gamma_g' = \frac{\Gamma_1 \Gamma_t^2 + \Gamma_{t2}}{T^2 \Gamma_{t1} \Gamma_{t2} - T^2 T_t^2}.$$

With this adjustment no variation in M occurs as the short is moved.

- 3) The meter reading is now directly proportional to $|\Gamma_L|$. That is $M = K|\Gamma_L|$. The meter reading is adjusted to the reference value of unity by adjustment of a gain control. If now an unknown load is connected to the system, the meter will accurately measure the magnitude of its reflection coefficient. In a practical case it may be necessary to apply corrections to the meter readings to take account of small deviations of the detector law from the meter law.

PHASE MEASUREMENT OF A SMALL REFLECTION

By a variation of the method just described, it is possible to measure quite accurately the phase angle of a small reflection.

Fig. 5 shows the flow graph of the setup required for phase measurement. A third slide-screw tuner is included just ahead of the load. Otherwise the flow graph is identical to Fig. 4. The probe itself is represented as a shunt discontinuity Γ_p distant θ and ϕ from the tuner's two ports. The small residual reflections at the load end of the tuner are represented by the general two-port flow graph with coefficients Γ_1' , Γ_2' , T' , while the residual reflections at the other end are considered lumped within the ports of the previous tuner network.

Assuming that Γ_1 , Γ_2 , D , Γ_{t1} , Γ_{t2} , Γ_1' , Γ_2' are all small compared to unity, the solution for the meter reading M is:

$$M \cong CkE' \frac{D + T\Gamma_{t1} + TTt^2\Gamma_p e^{-2j\theta} + TTt^2e^{-2j(\theta+\phi)}(1 + 2\Gamma_p)\Gamma_1' + TTt^2T'^2(1 + 2\Gamma_p)e^{-2j(\theta+\phi)}\Gamma_L}{1 - \sum L(1) + \sum L(2) \dots}.$$

The probe is first removed making Γ_p zero, and $|\Gamma_L|$ is then measured by the previous procedure of Fig. 4. During this procedure the tuner coefficient Γ_{t1} will have been adjusted to bring to zero the sum of all terms in the numerator not involving Γ_L . That is,

$$D + T\Gamma_{t1} + TTt^2e^{-2j(\theta+\phi)}\Gamma_1' = 0.$$

The probe is now inserted and adjusted in depth and position until M is zero. The result is

$$\Gamma_L = \frac{-\Gamma_p e^{+2j\phi}}{1 + 2\Gamma_p} \left(\frac{1}{T'^2} + \frac{2\Gamma_1' e^{-2j\phi}}{T'^2} \right)$$

or

$$\Gamma_L \cong \frac{-\Gamma_p e^{+2j\phi}}{1 + 2\Gamma_p} (1 - 2t' + 2\Gamma_1' e^{-2j\phi}).$$

Where t' is the small deviation of T' from unity, ($T' = 1 + t'$). For small reflections (less than 0.2) the probe can be considered a pure shunt susceptance, in which case,

$$\Gamma_p = \frac{-jB_p}{2 + jB_p}$$

and

$$\Gamma_L \cong \frac{jB_p}{2 - jB_p} e^{2j\phi} (1 - 2t' + 2\Gamma_1' e^{-2j\phi}).$$

Except for the error term in brackets, Γ_L is the complex conjugate of Γ_p transformed through the line length ϕ . The phase angle can be determined by using a Smith Chart or, solving for B_p in terms of $|\Gamma_L|$,

$$|\Gamma_L| = 2\phi - \tan^{-1} \left\{ \frac{(1 - |\Gamma_L|^2)^{1/2}}{|\Gamma_L|} \right\}.$$

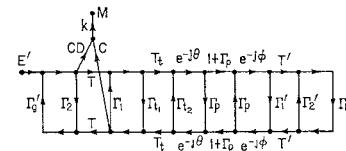


Fig. 5—Phase measurement with single coupler reflectometer.

The maximum error in the measurement of the phase angle is then $\sin^{-1}(2|t'| + 2|\Gamma_1'|)$. If the tuner probe is not lossless but has a small shunt conductance G_p associated with it the maximum error becomes

$$\sin^{-1} \left(2|t'| + 2|\Gamma_1'| + \frac{G_p}{B_p} \right).$$

However, if the greatest accuracy is desired, the probe conductance could be measured and the phase angle corrected. If a slotted line with the same residual dis-

continuities as the tuner is used, the effective load reflection measured would be

$$\Gamma_L \left(1 + 2t' + \frac{\Gamma_1'}{\Gamma_L} \right).$$

There is both a magnitude and a phase error. The error in the phase angle would be of the order of

$$\sin^{-1} \left(2|t'| + \left| \frac{\Gamma_1'}{\Gamma_L} \right| \right).$$

As an example, consider a case where a phase measurement is required of a load whose reflection coefficient has a magnitude of 0.03 (SWR = 1.06). Suppose the tuner used for the measurement has a residual reflection of 0.01. Then t' and Γ_1' could be of the order of 0.01. G_p/B_p can be of the order of 0.05. The maximum phase error would be $\sin^{-1}(0.09)$ or 5 degrees. If a slotted line with a residual reflection of 0.01 were used to measure the phase angle of the load reflection, the maximum error would be of the order of $\sin^{-1}(0.35)$, or 20 degrees, not to mention the error which could occur through inability to locate accurately the minimum in the standing wave.

CONCLUSION

The chief advantages of flow graph over matrix algebra in solving cascaded networks are the convenient pictorial representation and the painless method of proceeding directly to the solution with approximations being obvious in the process. The flow graph method is particularly useful in analyzing a measurement technique to determine residual error magnitudes.

The reflectometer techniques described are mainly applicable to the measurement of small reflection loads.

The use of tuners in the magnitude measurement results in a cancellation of residual error signals. In the phase measuring method, the residual error signals are merely depressed since a further probe insertion is required to make the measurement after "flattening" the system. This depression becomes important when the residual reflections in the system are of the same order of magnitude as the reflection to be measured.

APPENDIX I

THE SLIDE-SCREW TUNER

The slide-screw tuner consists of a probe of adjustable penetration projecting into a line through a slot along which its position can be adjusted. The probe itself can be regarded as a purely shunt discontinuity. In addition, there are fixed discontinuities at the ends of the slot and at the connectors or flange joints. It is desirable to lump all the fixed discontinuities at the two ports of the network. To show that this can be done, consider the flow graph of a shunt discontinuity followed by a length of lossless line as in Fig. 6(a).

Here β is the electrical length of the line section and Γ is the reflection coefficient of the discontinuity when backed up with a matched load. The discontinuity can be transferred to the other port as shown in Fig. 6(b).

In either case the scattering coefficients are

$$S_{11} = \Gamma, \quad S_{22} = \Gamma e^{-2i\beta}, \quad S_{12} = S_{21} = (1 + \Gamma)e^{-i\beta}.$$

If a further discontinuity Γ' is present at the right-hand port, the two can be lumped together and described by the flow graph of Fig. 6(c) where,

$$\begin{aligned} \Gamma_1 &= \frac{\Gamma e^{+2i\beta} + \Gamma' + 2\Gamma\Gamma'}{1 - \Gamma\Gamma'e^{-2i\beta}}, \\ \Gamma_2 &= \frac{\Gamma' + \Gamma e^{-2i\beta} + 2\Gamma\Gamma'e^{-2i\beta}}{1 - \Gamma\Gamma'e^{-2i\beta}}, \\ T &= \frac{e^{-i\beta}(1 + \Gamma)(1 + \Gamma')}{1 - \Gamma\Gamma'e^{-2i\beta}}. \end{aligned}$$

For small reflections, the tuner probe is a lossless shunt discontinuity and is equivalent to a shunt capacitive susceptance. The relationship between normalized susceptance B_p and probe reflection coefficient Γ_p is

$$\Gamma_p = \frac{-jB_p}{2 + jB_p}.$$

The complete flow graph of a slide-screw tuner is shown in Fig. 6(d) where all the fixed reflections beyond the probe are now lumped at the two ports.

It is desirable to represent the tuner by a two-port flow graph with three coefficients. In order that this be

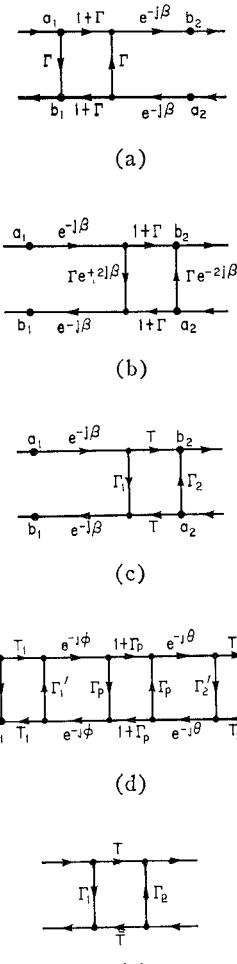


Fig. 6—(a) Shunt discontinuity followed by a length of lossless line. (b) Equivalent discontinuity referred to the other port. (c) Additional discontinuity included at the port. (d) Complete flow graph of the slide-screw tuner. (e) Equivalent side-screw tuner flow graph.

useful, however, it is necessary to show that the S_{11} and S_{22} coefficients can be made equal to any arbitrary value by proper adjustment of Γ_p and θ . This can be done in two steps. Consider first the case with no discontinuity at port 1. The S_{11} coefficient is then

$$\frac{\Gamma_p e^{-2i\phi} + \Gamma'_2 e^{-2i(\theta+\phi)} + 2\Gamma_p \Gamma'_2 e^{-2i(\theta+\phi)}}{1 - \Gamma_p \Gamma'_2 e^{-2i\theta}}.$$

Consider the possibility of making this some arbitrary value k . This is a simple problem to solve using a Smith Chart. One would start with a reflection coefficient Γ'_2 at port 2 and move toward the generator until reaching a point at which the reflection $\Gamma'_2 e^{-2i\theta}$ and the reflection $k e^{+2i\theta}$ were represented on the chart by admittances with the same conductance value. The probe would be inserted at this point until its susceptance equalled the difference between the susceptances at the points representing the two reflections.

Stated analytically these conditions are

$$\begin{aligned}\Gamma_p &= \frac{-jB_p}{2+jB_p}, \\ ke^{+2j\phi} &= \frac{1-G-jB_k}{1+G+jB_k}, \\ \Gamma'_2 e^{-2j\theta} &= \frac{1-G-jB'}{1+G+jB'}, \\ B_p &= B_k - B' \quad [B_p \text{ positive}].\end{aligned}$$

Substitution of these conditions in the expression for S_{11} above does give the result $S_{11}=k$. Since the S_{11} coefficient can be made equal to any arbitrary value k when the fixed port-1 reflection is absent, it is obvious that the S_{11} coefficient for the complete system can be made

equal to any arbitrary value a . The value is

$$a = \frac{\Gamma_1''(1-k\Gamma_1') + T_1^2 k}{1-k\Gamma_1'},$$

or

$$k = \frac{\Gamma_1'' - a}{\Gamma_1''\Gamma_1' - T_1^2 - \Gamma_1'a}.$$

The slide-screw tuner can now be represented as a two-port device with three coefficients as shown in Fig. 6(e), and we can conclude that Γ_1 or Γ_2 can be made any arbitrary value by adjustment of the probe.

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Stepped Transformers for Partially Filled Transmission Lines*

D. J. SULLIVAN† AND D. A. PARKES†

Summary—In recent years, partially-filled transmission lines have been used to improve the characteristics of various ferrite and garnet devices. This paper presents a generalized outline for determining the approximate effective guide wavelength and characteristic impedance of two types of (dielectric-loaded) partially-filled transmission line. The results are used to determine the geometries required for the design of optimum stepped transmission line transformers. The stepped transitions are designed to yield a Tchebycheff-type response for any given bandwidth. The measured results for stepped transitions in partially filled coaxial line and partially filled double-ridge waveguide are presented. The data are found to approximate the theory closely.

I. INTRODUCTION

Dielectric-loading techniques^{1,2} are frequently used to improve the characteristics of certain ferrite and garnet devices. It has been shown that one method of obtaining a nonreciprocal device in conventional coaxial or strip transmission line is to distort the dominant (TEM or Quasi-TEM) mode by use

of a dielectric material.^{3,4} The nonreciprocal characteristics of double-ridge waveguide components are also often improved by supplementing the ferrite or garnet with a dielectric material.⁵ The addition of this dielectric material changes the characteristic impedance of the transmission line, and this in turn introduces the problem of matching. Cohn has shown that for a given number of steps a Tchebycheff stepped transformer design will give the minimum possible VSWR for a specified bandwidth.⁶ Three stepped transitions, in partially filled transmission line, are shown in Fig. 1. The use of a stepped transition will normally 1) substantially reduce the inherent VSWR of a device, 2) enable a specific unit to be made considerably shorter or 3) result in a compromise between the two.

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† Sperry Microwave Electronics Co., Clearwater, Fla.

¹ P. H. Vartanian, J. L. Melchor, and W. P. Ayres, "Broadbanding ferrite microwave isolators," 1956 IRE NATIONAL CONVENTION RECORD, pt. 5, pp. 79-83.

² E. A. Ohm, "A broadband microwave circulator," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-4, pp. 210-217; October, 1956.

³ B. J. Duncan, L. Swern, K. Tomiyasu, and J. Hannwacker, "Design considerations for broadband ferrite coaxial line isolators," PROC. IRE, vol. 45, pp. 483-490; April, 1957.

⁴ D. Flerl and G. Hanley, "Nonreciprocity of dielectric loaded TEM mode transmission lines," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-7, pp. 23-27; January, 1959.

⁵ E. Grimes, D. Bartholomew, D. Scott, and S. Sloan, "Broadband ridge waveguide ferrite devices," presented at the IRE National Symposium on Microwave Theory and Techniques, Harvard University, Cambridge, Mass.; June 1-3, 1959.

⁶ S. B. Cohn, "Optimum design of stepped transmission line transformers," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-3, pp. 16-21; April, 1955.